Transformers

Plan

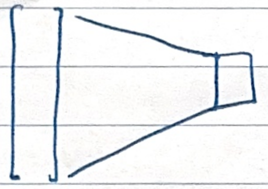
Review
Self-Attention
Cross-Attention
Positional Encoding

Logistics

check in
scribe
zoom

Review

Motivation: words as vectors



one-hot encoding

$f: \mathbb{R}^{|V|} \to \mathbb{R}^d$

(x, x^+) close $\Rightarrow f(x) \cdot f(x^+)$ large

(x, x^-) far $\Rightarrow f(x) \cdot f(x^-)$ small

Contrastive learning $\subseteq$ unsupervised learning

Principal Component Analysis

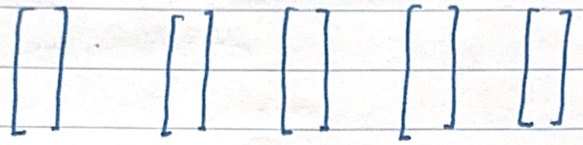
capture variation of ~~eigenvectors~~ data via eigenvectors

$$X^T X = \sum_{i=1}^{r} \lambda_i v^{(i)} v^{(i)T} \quad \text{for} \quad v^{(i)} \cdot v^{(j)} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{else} \end{cases}$$

$$\max_{v : \|v\|_2 = 1} \|Xv\|_2^2 \leftarrow v^T X^T X v \leftarrow \sum_{i=1}^{n} \lambda_i [v^T v^{(i)}]^2$$

Motivation: sentences as vectors

"Vermont is chilly and beautiful"



How can we understand sequences of vectors?

↳ Recurrent networks

↳ LSTM

Attention! (self first) $X \in \mathbb{R}^{n \times d}$

Goal: Combine similar words/tokens

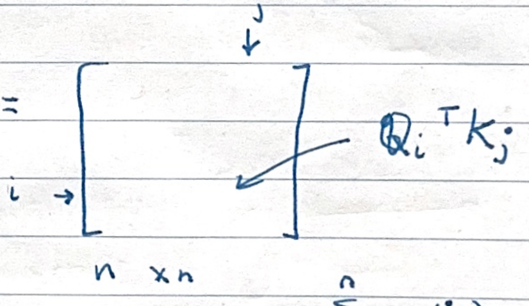
Queries: $W^{(Q)} X = Q$ $\quad W^{(Q)} \in \mathbb{R}^{r \times d}$

Keys: $W^{(K)} X = K$

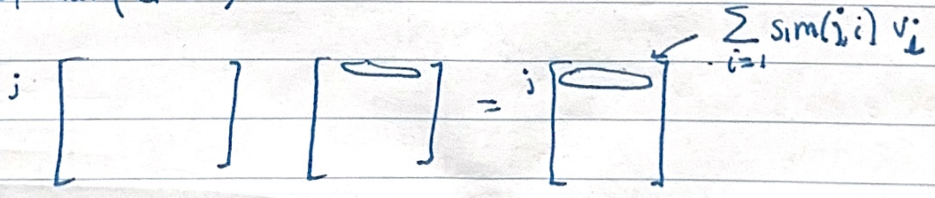
Values: $W^{(V)} X = V$ $\quad W^{(V)} \in \mathbb{R}^{d \times d}$

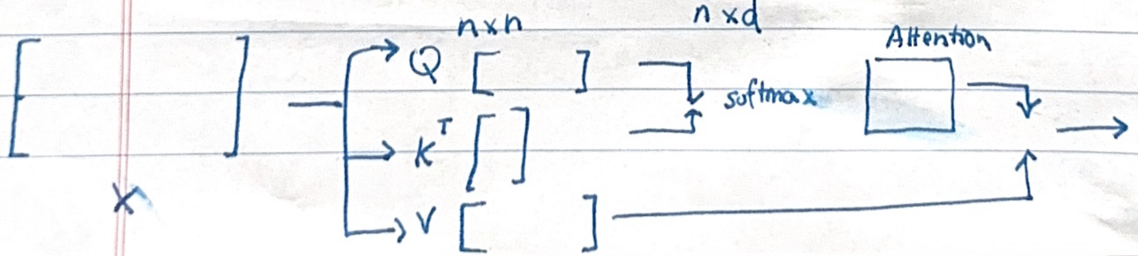
attention : $\text{softmax}(Q K^T) =$

to rows

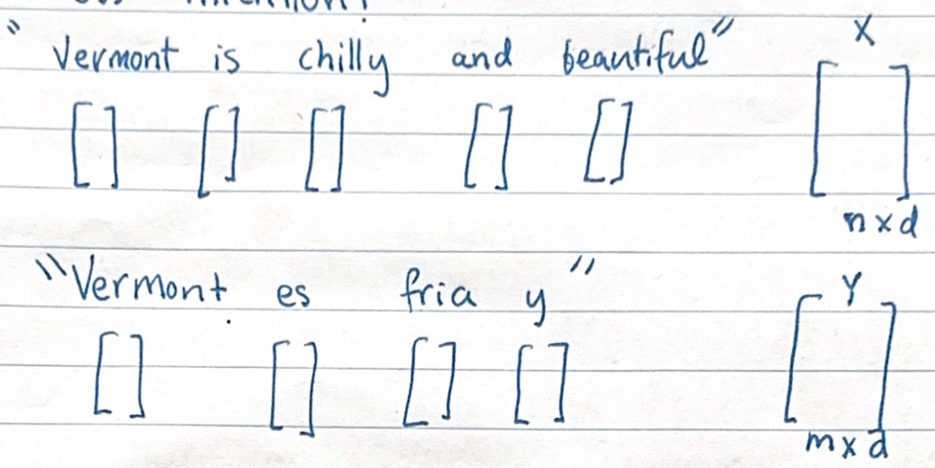


result: $\text{softmax}(Q K^T) V$





Cross- Attention!



"Vermont is chilly and beautiful"

"Vermont es fria y"

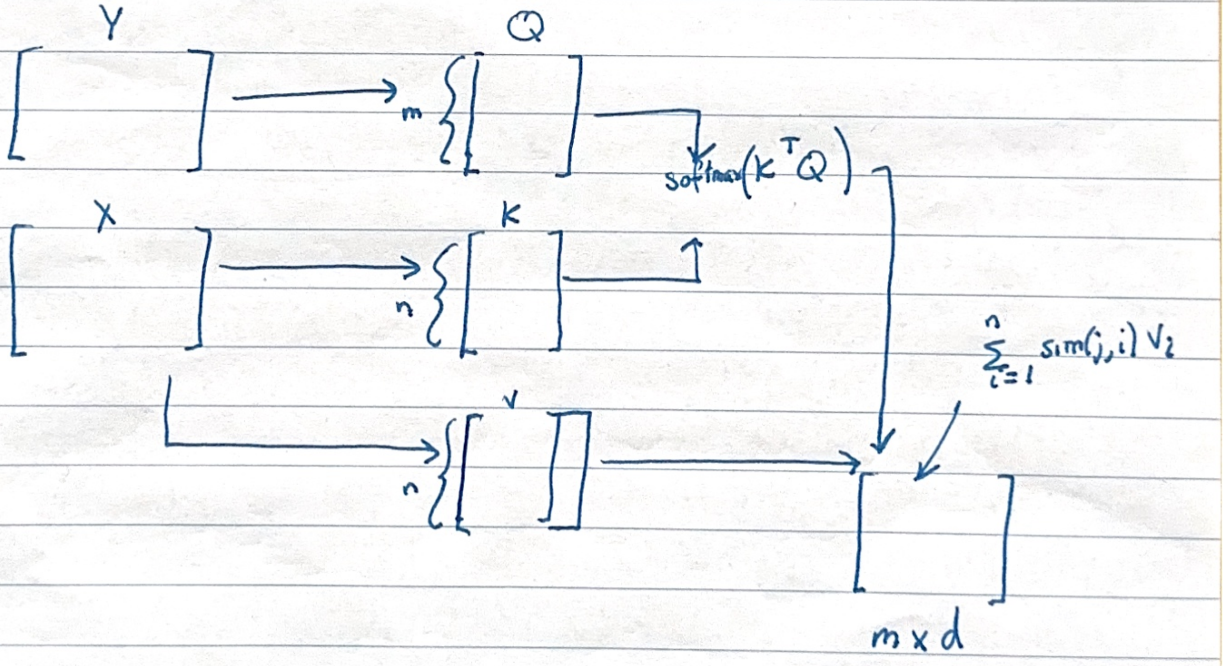
Goal: Represent sequence as linear combo of another

$X \in \mathbb{R}^{n \times d}$

Queries: $W^{(Q)} \cancel{X} = Q$ $\quad W^{(Q)} \in \mathbb{R}^{r \times d}$

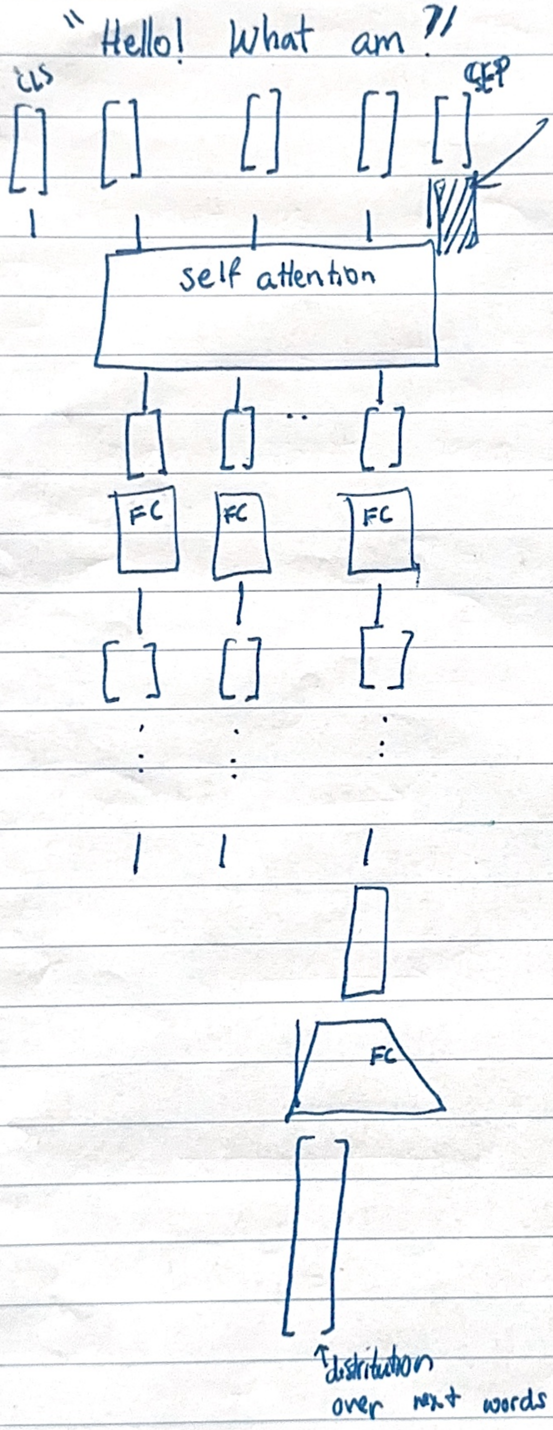
$W^{(K)} X = K$ $\quad W^{(K)} \in \mathbb{R}^{r \times d}$

$W^{(V)} X = V$ $\quad W^{(V)} \in \mathbb{R}^{d \times d}$



Make upwards direction?

Large Language Models

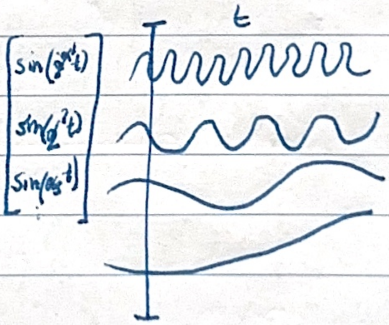


Positional Encoding?

We represent time as 11:24am Tuesday, Jan 14, 2025 rather than 1,065,066,880 min since 0 BC

↳ min captures schedule
↳ hour captures time of day
↳ day captures schedule
↳ date captures schedule
↳ month captures time of year
↳ year captures years passed

"Hello! What am I?"
0 1 2 = t


t
$\sin(g^{1}t)$
$\sin(g^{2}t)$
$\sin(a_{s}t)$